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Editorial

AN ASPECT OF PARTICLES' SPATIAL COMPETITION

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Abstract

The branching random walk on integer lattices is shown to be monotone decreasing in ascending direction (MDAD) as a corollary of an observation in regard to the lineage of particles from antisymmetric initial states, and a related property of percolation paths on the usual oriented lattices is also given.

Key words: Stochastic domination; symmetry; monotonicity; spatial branching processes; oriented percolation

Let Z^d be the set of all *d*-vectors of integers endowed with the natural preorder induced by coordinate-wise modulus, denoted by ≤. The following criterion for a collection *X* of random variables ($X(z); z \in Z^d$) is introduced. For all $z, w \in Z^d$, if $z \leq w$ then $X(z) \succ_{st.} X(w)$, where $_{st.}$ denotes stochastic domination. The basic branching random walk (BRW) on Z^d started from a particle at the origin is shown to satisfy this criterion at all times. For terminology and background on the BRW we refer the reader to I.1 in [5].
The method of proof here is by deduction from a spatial competition property for the discrete-time counterpart of the process done in §1, whereas, in §2, a property of that type for oriented percolation paths is shown.

1. a) A population of particles inhabiting Z, the set of sites positioned at the integers, evolves in generations in discrete-time according to the following, parameter *p*, rules. Every particle begets another at each nearest site with probability *p* at the instant of its demise. All births are independent of one another.
Started from one progenitor at the origin, {0}, note that the only feasible (inhabited with non-zero probability) sites are the even at odd times and the odd at even times.

Proposition 1. *At all times for any duad of nearest feasible sites the cardinality of particles at the one nearest to the origin stochastically dominates the other.*

The Proposition above comes from translation invariance and taking *C* in the next statement to be singletons.
A. *At all time and on any subset C of* {0,1,2,...} *the distribution of particles descending from* {−1} *equals that of those descending from* {1} *and visited* {0} *at some time prior or equal to that time*.
The statement in A is a consequence of that in A^0 below by noting that particles with non-negative spatial coordinate at any given time have to have visited at least once site {0} at some time prior or equal to that time.
A^0. *At all time the distribution of particles descending from* {1} *and visited site* {0} *prior to that time equals that of those from* {−1} *instead.*
We elaborate on the idea of "switching the spin upon collisions at the symmetry axis" coupling, thus, proving A^0. Particles descending from {−1} are tagged as *α*-particles for as long as they occupy sites with negative spatial coordinate, while those descending from {1} are tagged as *β*-particles for as long as they occupy sites with positive one. Further, particles born at {0} from *α*-particles are tagged as *γ*-particles, while those born from *β*-particles are tagged as *δ*-particles. Coupling *α*-particles with *β*-particles antithetically imposes that births of *γ*-particles and *δ*particles are simultaneous, which thus permits coupling them upon birth identically.

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From the Proposition above the result claimed in the introduction for $d = 1$ follows by drawing upon known approximation methods, on which we suffice it to note that the one proposed in [2] is used here in an essential way. Further, a little thought shows that the Proposition above is extended for the process corresponding to a) on Z^d analogously. From this the result claimed follows in all generality since for each d the extension of the method refer to is carried out straightforwardly.
Finally, it is worth remarking here that the resultant dismantling from consideration of the discrete time counterpart of the process had been instructive throughout this section.
2. Let L be the usual oriented percolation lattice, that is, the set of space-time points (y,k) ⍰ Z^2, $y + k$ *even*, $k \geq 0$, with adjunct points, $(y - 1,k + 1)$ and $(y + 1,k + 1)$. By bond percolation retaining parameter p it is understood that every bond is declared independently open with probability p, and as usual a path is defined as a sequence of consecutive adjunct points that includes exactly one at each time. The next statement is better illustrated by considering the embedded process: b) The population of particles evolves on Z as in a), but births from the same site to each nearest site are identically dependent. To see the connection note that the cardinality of open paths to a site in the former is the number of particles in the latter for appropriately similar initial conditions.
Note that the next statement is in the spirit of that in A^0 above and, remark that it will be seen by the proof that extensions for the process on Z^d analogously hold.

B. At all time n the distribution of particles descending from {1} and visited site {0} at any fixed time m, m ≤ n, equals that of those from {−1} instead.

Let L^0 denote L shifted to odd coordinates instead. Let further T_n be the subset of L^0 whose points are delimited within the isosceles trapezoid, vertices the points $(-1,0)$, $(1,0)$ and $(-1 - n,n)$, $(1 + n,n)$, for which, note that, $S_n := \{0\}\times\{0,1,\ldots,n\}$ is an axis of symmetry. By symmetry of points about S_n each point $(-y,k)$, $y \geq 1$, has a unique counterpart mirror image (y,k), while points of S_n we regard as the mirror image of themselves. Accordingly, the mirror image of every pair of neighbouring points is regarded to be the pair of their mirroring points.
The following idea of ”rotating the sample point” coupling proves B. Consider a realization of bond percolation retaining parameter p on T_n. Consider the operation of interchanging the value of the Bernoulli r.v. assigned to each pair of adjunct points with that associated to the pair's mirror image prior or equal to time m. Note that the operation results in dissection of paths passing through $(0,m)$ at that point as well as in their reflection about the symmetry axis prior to that time. Since further this operation defines a one-to-one correspondence and preserves the total number of bonds present the proof is completed.
Postscript. Literature related to the MDAD property is as follows. For the basic contact process started from the origin on Z, this is shown in [1], see also [2, 3]. Regarding highly subcritical bond percolation on Z^d, MDAD for sites on one axis is shown in [6]. For a rigorous proof of the MDAD for the BRW on Z^d, see [4], Lemma 11; Proposition 1 was shown independently by the author in [7].

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